The North Norwich Cluster Calculations Policy - Appendix

Progression of Understanding in Formal Methods of Calculation

Information and Guidance on the Teaching of Formal Methods of Calculation.

The thinking behind this appendix can be best described by the following extracts taken from *Understanding Mathematics for Young Children.* Haylock and Cockburn (2013). Pg. 185:

> ***'…there is no case for introducing children to vertical layouts before they have developed a basic understanding of place value.'***
>
> ***'Early introduction of vertical layout can be positively harmful.'***
>
> ***'Mental strategies based on visual or pictorial images, such as movements on the hundred square or along an empty number line, should be the major priority for the development of confidence and success in handling numbers, definitely for children under the age of 9.'***

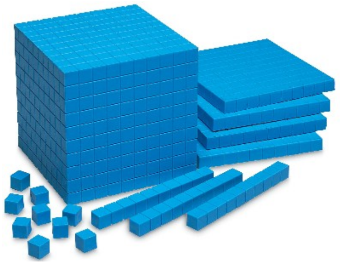
With these considerations in mind, the teaching of formal methods of calculation should **<u>only</u>** be taught under the following circumstances:

- From the summer term of year 5 onwards – not before.
- The child has shown a secure understanding of informal methods of calculation.
- The child has a secure understanding of their times tables up to 10x10.
- The methods are taught as an alternative to informal methods, not as the default method.
- If a child becomes confused by the formal algorithms, then revert back to the progression within the calculations policy to ensure that they are confident in calculation.

If there are any questions regarding this appendix,
please seek guidance from the Subject Leader for Mathematics.

Formal Algorithms for...Addition

The important considerations when teaching the formal algorithms for addition centre on the understanding of place value. To ensure that understanding is not lost to a simple process of subtracting the digits that are sitting on top of each other. To scaffold this understanding, teachers should plan to use concrete representations such as 'Base 10 / Dienes blocks' or money (1ps, 10ps and £1s).



This represents a step on from the partitioning method of addition as it is the first time that children will see a vertical layout.

For calculating 372 + 247 =

This stage should not be rushed as children need to comprehend that...

372 + 247 = 300 + 200 + 70 + 40 + 2 + 7 = 500 + 110 + 9 = 619

Stage 1:

300 + 70 + 2
200 + 40 + 7

500 + 110 + 9 = 619

At this stage, the children will be inclined to add the hundreds first, then the tens before adding units. As this will have been the order of calculation when they have added by partitioning. When it is appropriate, the teacher should move them towards adding the units first, so that they are prepared for stages 2 and 3.

Formal Algorithms for…**Addition**

Stage 2:

```
    3 7 2
  + 2 4 7
  -------
        9    (2+7)
    1 1 0    (70 + 40)
    5 0 0    (300 + 200)
  -------
    6 1 9
```

Stage 2 builds on the understanding that has been maintained in **stage 1**. This is the first time that the algorithm turns from a partitioning based calculation to one that could be 'digit based' . It is very easy to forget that in the stage 2 calculation , you do not calculate 7 + 4, you calculate 70 + 40. This is why it is important to write calculation in the brackets next to the partial sums.

The next developmental step is to do the same process, but without the calculations in the brackets. If the children make conceptual errors, then they should be taken back to **stage 1**.

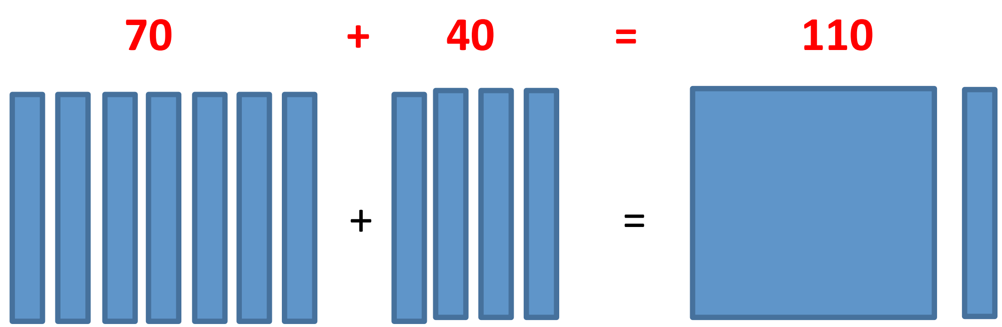
The next step is to remove the bracketed annotations as the calculation becomes more abstract.

Stage 3:

```
    3 7 2
  + 2 4 7
  -------
    6 1 9
  -------
     1
```

The conceptual jump that children need to make is in the 'carrying' of 10 from one column to the next. This is best illustrated by using 'Base 10' resources (below).

Using Base 10



Formal Algorithms for…

Subtraction

Subtraction is the number operation in which children tend to make the most 'little mistakes' in their calculation. Decomposition as a method (**stage 3**) can be taught for understanding, however it is often taught as a set of procedures to reach an answer.

By following **stages 1 to 3**, and by explaining the reasoning behind each stage, you can ensure that the children understand what they are calculating when they get to **stage 3**. The key point of confusion comes when children are required to 'borrow 10' from the column to the left. This needs to be explained by following the concrete-pictorial-abstract model to help the children develop conceptual understanding.

For calculating 448 – 267 =

Stage 1:

```
  400 + 40 + 8
- 200 + 60 + 7
  ____________
  200 – 20 + 1   = 181
  ____________
```

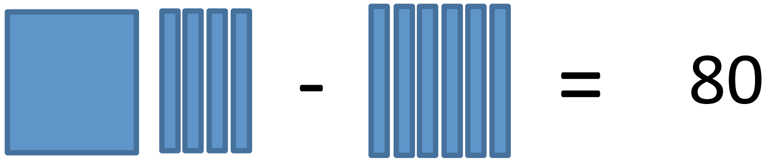
As with stage 1 of addition, the children are encouraged to see the calculation as a series of subtractions following the partitioning of the number into hundreds, tens and units.

The potential misconception comes when the children are asked to perform the calculation 40 – 60. The temptation will be to reverse the calculation, or to omit the – sign from the answer.

When finding the answer to the calculation, the children will need to have a secure understanding of the meaning of the + and – signs so that they can complete the calculation 200-20+1.

Formal Algorithms for…

Subtraction



Stage 2:

300 140
~~400~~ + ~~40~~ + 8
\- 200 + 60 + 7

100 + 80 + 1 = 181

The understanding required for full decomposition can be developed by representing the 'borrowing' of 100 from the 400 with 'Base 10' resources. Adding the 100 to the 40 to make 140, means that you can now subtract 60 from 140 to equal 80.

This process means that you don't end up with a negative number at the bottom and that the jump to decomposition is smaller.

This is due to retaining the understanding that when you 'borrow' you are 'borrowing' 10 or 100, not 1. This is consolidated by crossing out the 40, and writing 140 above it, rather than just tagging on a 1.

Stage 3:

3
~~4~~ 14 8
\- 2 6 7

1 8 1

In the example for **stage 3**, we have 'borrowed' the 100 from the hundreds column and added it to the tens column. Using 'Base 10' resources, you can highlight that the revised calculation shows the calculations: 8-7, **14 (tens) – 6 (tens),** 3 (hundreds) – 2 (hundreds). For understanding, the part to highlight is the *14 tens – 6 tens*, as this is where the misconception can be. (See diagram in **stage 2**)

Formal Algorithms for…Multiplication

When choosing the methods for calculating with multiplication, it is important to consider how the method helps you to understand the distributivity and commutativity of multiplication and whether the chosen method aids the calculation itself. This is the reason for placing an emphasis on the methods shown in the main body of the calculations policy.

When using the formal algorithms for multiplication, identifying the distributivity and commutativity is not always possible. For this reason, it is important that the children understand what they are calculating and why at each stage of the algorithm.

There are different methods for calculating multiplication (such as the 'Lattice method') which are not shown in this appendix. These are the methods that the children will need to know for the Key Stage 2 assessments.

Short Multiplication: ***342 x 7 =***

```
   3 4 2
x      7
--------
     1 4   (2x7)
   2 8 0   (40x7)
 2 1 0 0   (300x7)
--------
 2 3 9 4
```

This method highlights the distributive law of multiplication as, like the grid method, you have to separate the component parts of the number to perform the calculation. The misconceptions can occur when 40x7 becomes 4x7. The place value of the digits needs to be preserved.

```
   3 4 2
x      7
--------
 2 3 9 4
--------
   2 1
```

As addition, short multiplication should start in an expanded form, before progressing to the compact version.

Formal Algorithms for...**Multiplication**

146 x 84 =

Long Multiplication - Step 1:

	100	40	6
80	8000	3200	480
4	400	160	24

```
    1 4 6
x     8 4
---------
      2 4   (6x4)
    1 6 0   (40x4)
    4 0 0   (100x4)
    4 8 0   (6x80)
  3 2 0 0   (40x80)
  8 0 0 0   (100x80)
---------
1 2 2 6 4
---------
```

For long multiplication, start with the conceptual understanding that is provided by the 'grid method' to scaffold the step towards the formal algorithm. Each method highlights the distributive law, especially when you start with the expanded form of long multiplication in **step 1**.

Long Multilication - Step 2:

```
    1 4 6
  x   8 4
---------
    5 8 4
     1 2
1 1 6 8 0
   3 4
---------
1 2 2 6 4
   1 1
```

As with short multiplication, the place value of each digit can be lost if 6x80 is read as 6x8. Misconceptions are formed when people say, 'just write a zero on the second line'. The reason for this is that we are multiplying by 80 rather than by 8. Children should be encouraged to say that the calculation is 6x80 and use their knowledge of the number fact 6x8 to help them find the product 480.

Formal Algorithms for…**Division**

As with multiplication, when choosing calculation methods for division, there are certain considerations to take into account. Does the method show that division is not commutative? Does it highlight that you can swap the divisor and the quotient, but you cannot swap the divisor and the dividend? Does the method aid calculation? Using arrays and the number line do highlight these issues, however with the bus stop method, it can be less clear.

Children should know how the bus stop methods for short and long division work, but are not expected to use them as their default method of calculation. To promote understanding of division and the calculation that is being performed, children should be encouraged to use the number line when dividing.

These methods should be learned so that children can answer questions in the Key Stage 2 assessments.

Short Division ***432 ÷ 5***

```
      8  6  r2
   ┌──────────
5  │ 4  3 ³2
```

Answer = 86 r 2 or $86\frac{2}{5}$ or 86.4

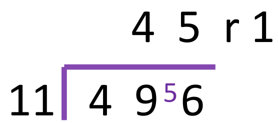
432 ÷ 5 in isolation is a difficult calculation, mentally partitioning the number by place value in short division can help. You could calculate 400 ÷ 5 = 80, however this is unhelpful as recording the 80 would require writing the 8 in the tens column and ignoring the 0, which could be confusing. For this reason, calculate 43 ÷ 5, which gives 8, and then carry the 3 to make 32. This can then be divided by 5 to equal the 6 with 2 remainders.

The remainder can be left as it is, or divided by 5 to give $\frac{2}{5}$ or 0.4.

Formal Algorithms for...Division

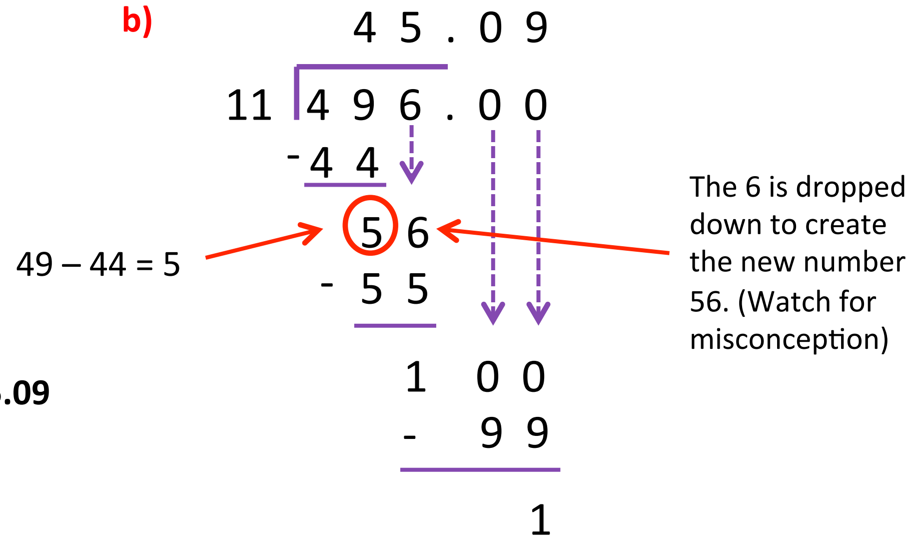
<u>**Long Division**</u> ***496 ÷ 11***

a)



Answer: 45 remainder 1 or 45 $\frac{1}{11}$ or 45.09

b)



Long division is a method for dividing a 2 or 3 digit number by a number of at least 2 digits. Example **a)** shows that it is possible to use short division for this type of calculation to derive the answer 45r1 or 45$\frac{1}{11}$. To gain greater precision including decimal places, long division is required. Example **b)** shows how the first two digits of the dividend (496) is divided by 11 (49÷11) to equal 44 with 5 left over. This is shown as the subtraction beneath the dividend. You then drop the 6 down to make the 5 into 56. (Potential misconception here due to 5 becoming 5 tens). This is then divided by 11 and the remainder recorded as before. As the remainder is now 1, you drop a zero down to make 10, then once more to make it 100 (See previous place value misconception). This process can be continued to add additional decimal places as required.